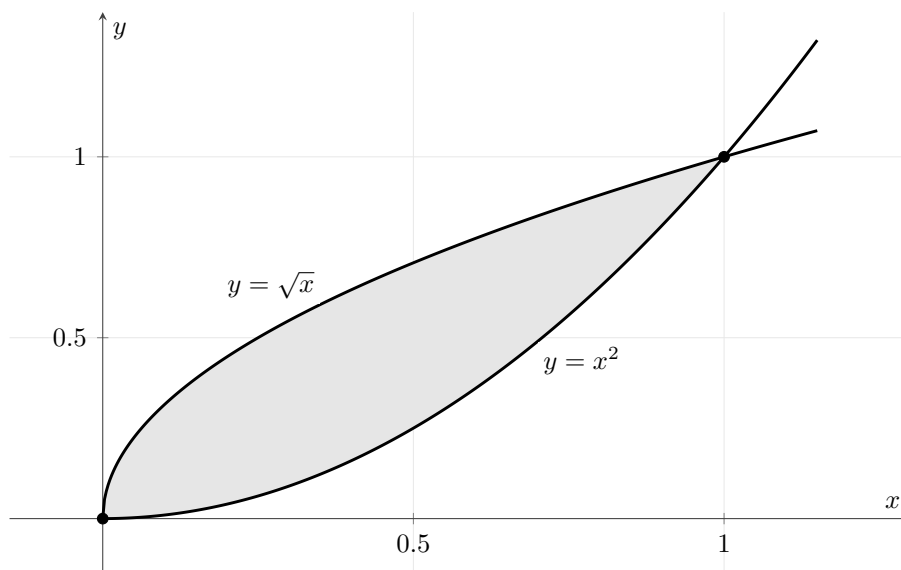


Question 5

Find the exact area of the region enclosed by the curves

$$y = \sqrt{x} \quad \text{and} \quad y = x^2,$$

between their points of intersection.



The shaded region is the area to be found.

Question 5 – Solution

1. Find the intersections.

The square root requires $x \geq 0$. Both sides are nonnegative, so squaring gives

$$\sqrt{x} = x^2 \implies x = x^4 \implies x(x^3 - 1) = 0.$$

The factor $x^3 - 1 = (x - 1)(x^2 + x + 1)$ has only one real zero, $x = 1$. The real solutions $x = 0$ and $x = 1$ both satisfy the original equation. The intersection points are $(0, 0)$ and $(1, 1)$.

2. Determine which curve is above the other.

For $0 < x < 1$, $x^{1/2} > x^2$. For example, at $x = 1/4$, $\sqrt{x} = 1/2$ while $x^2 = 1/16$.

3. Set up upper minus lower.

$$A = \int_0^1 (\sqrt{x} - x^2) dx.$$

4. Integrate each power.

Use $\int x^n dx = x^{n+1}/(n+1)$ for $n \neq -1$:

$$\int x^{1/2} dx = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}, \quad \int x^2 dx = \frac{x^3}{3}.$$

5. Apply the bounds and subtract.

$$\begin{aligned} A &= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3} \right]_0^1 \\ &= \left(\frac{2}{3} - \frac{1}{3} \right) - 0 = \boxed{\frac{1}{3}}. \end{aligned}$$