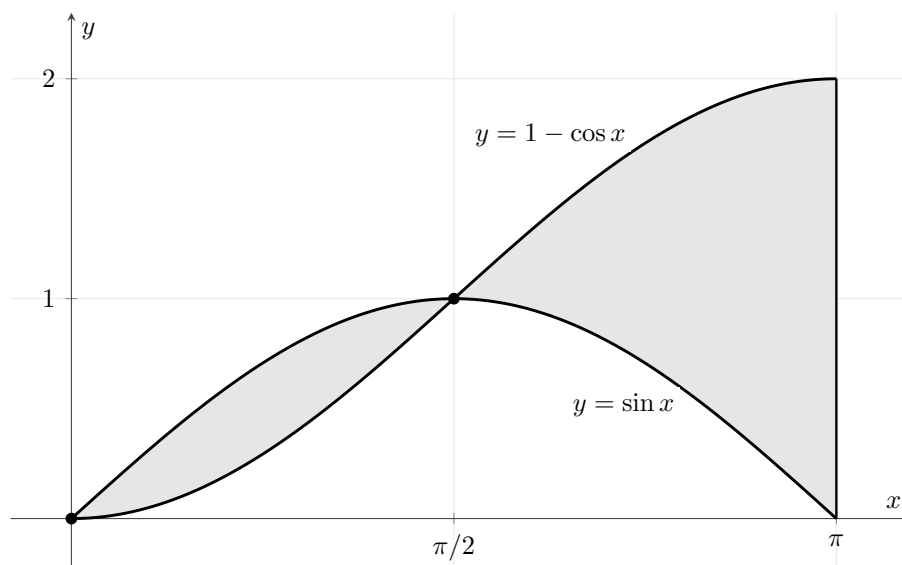


Question 3

Find the exact area of the region between the curves

$$y = \sin x \quad \text{and} \quad y = 1 - \cos x,$$

between $x = 0$ and $x = \pi$.



The shaded region is the area to be found.

Question 3 – Solution

1. Find where the curves cross on $[0, \pi]$.

Using half-angle identities,

$$\begin{aligned}\sin x = 1 - \cos x &\iff 2 \sin(x/2) (\cos(x/2) - \sin(x/2)) = 0 \\ &\iff x = 0 \quad \text{or} \quad x = \frac{\pi}{2}.\end{aligned}$$

2. Determine the upper curve on each interval.

On $(0, \pi/2)$, $\sin x > 1 - \cos x$; on $(\pi/2, \pi)$, the order reverses. Test $x = \pi/4$: $\sin x = \sqrt{2}/2 > 1 - \sqrt{2}/2$. At $x = 3\pi/4$, $1 - \cos x = 1 + \sqrt{2}/2 > \sqrt{2}/2$. The line $x = \pi$ closes the region on the right.

3. Set up the sum of the two areas.

$$A = \int_0^{\pi/2} (\sin x - 1 + \cos x) dx + \int_{\pi/2}^{\pi} (1 - \cos x - \sin x) dx.$$

4. Integrate each piece and apply its bounds.

$$A_1 = [-\cos x - x + \sin x]_0^{\pi/2} = 2 - \frac{\pi}{2},$$

$$A_2 = [x - \sin x + \cos x]_{\pi/2}^{\pi} = (\pi - 1) - \left(\frac{\pi}{2} - 1\right) = \frac{\pi}{2}.$$

5. Add the positive areas.

$$A = A_1 + A_2 = \left(2 - \frac{\pi}{2}\right) + \frac{\pi}{2} = \boxed{2}.$$