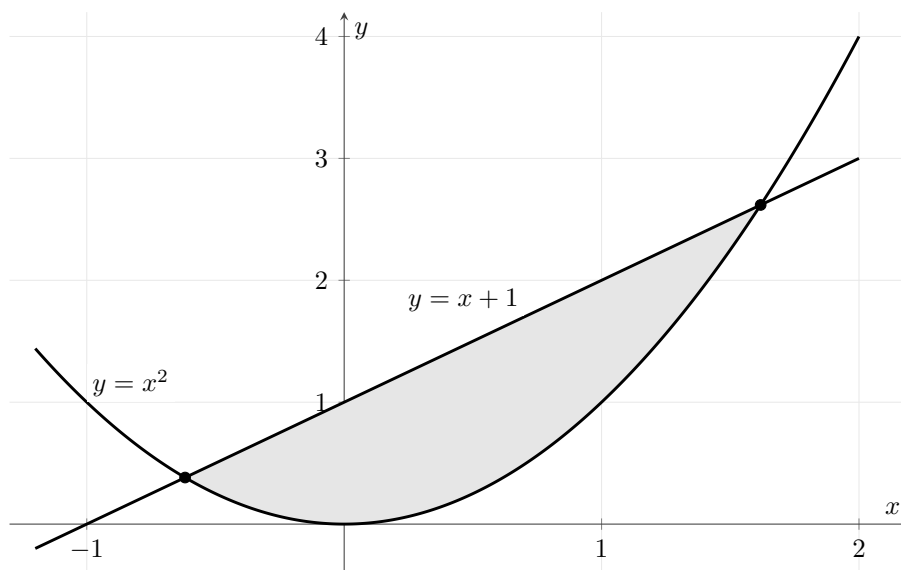


Question 2

Find the exact area of the region enclosed by the curves

$$y = x + 1 \quad \text{and} \quad y = x^2,$$

between their points of intersection.



The shaded region is the area to be found.

Question 2 – Solution

1. Find the intersections.

$$x^2 = x + 1 \implies x^2 - x - 1 = 0.$$

The quadratic formula gives $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$. Write the two roots as

$$a = \frac{1 - \sqrt{5}}{2}, \quad b = \frac{1 + \sqrt{5}}{2}.$$

2. Determine which curve is above the other.

Test $x = 0$: the line has height 1 and the parabola has height 0. Between the roots, $(x + 1) - x^2 \geq 0$, so the line is above the parabola.

3. Set up upper minus lower.

$$A = \int_a^b (-x^2 + x + 1) dx.$$

4. Complete the square to simplify the bounds.

Complete the square:

$$-x^2 + x + 1 = \frac{5}{4} - \left(x - \frac{1}{2}\right)^2.$$

Let $u = x - \frac{1}{2}$. The new bounds are $-\sqrt{5}/2$ and $\sqrt{5}/2$. Here $du = dx$. The integrand is even, so the two halves have equal area.

5. Integrate and simplify.

$$\begin{aligned} A &= 2 \int_0^{\sqrt{5}/2} \left(\frac{5}{4} - u^2 \right) du \\ &= 2 \left[\frac{5}{4}u - \frac{u^3}{3} \right]_0^{\sqrt{5}/2} \\ &= 2 \left(\frac{5\sqrt{5}}{8} - \frac{5\sqrt{5}}{24} \right) = \boxed{\frac{5\sqrt{5}}{6}}. \end{aligned}$$