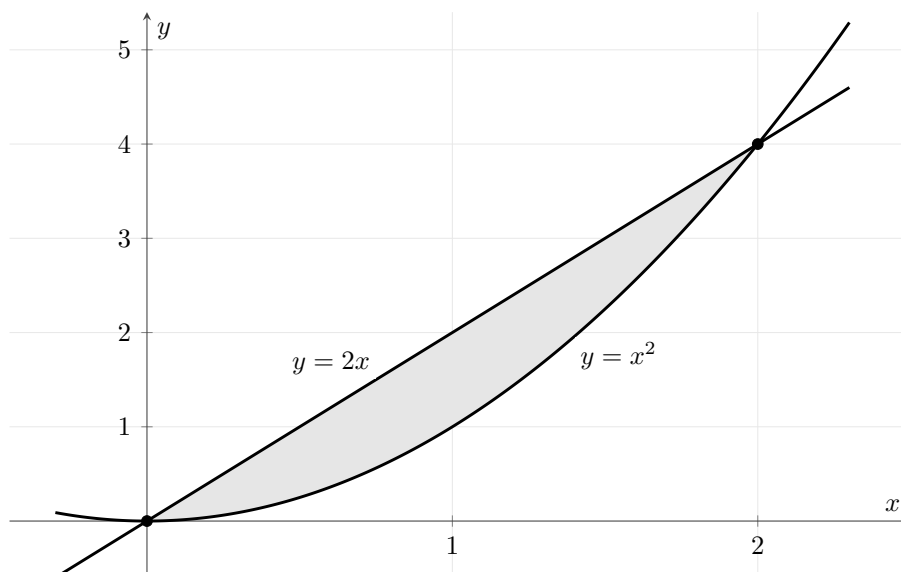


Question 1

Find the exact area of the region enclosed by the curves

$$y = x^2 \quad \text{and} \quad y = 2x,$$

between their points of intersection.



The shaded region is the area to be found.

Question 1 – Solution

1. Find the intersections.

$$x^2 = 2x \implies x(x - 2) = 0 \implies x = 0, 2.$$

Factoring gives $x = 0$ or $x - 2 = 0$. Substituting into $y = 2x$ gives $y = 0$ or $y = 4$. The intersection points are $(0, 0)$ and $(2, 4)$.

2. Determine which curve is above the other.

On $0 \leq x \leq 2$, $2x - x^2 = x(2 - x) \geq 0$, so the line is above the parabola.

3. Set up upper minus lower.

$$A = \int_0^2 (\text{upper} - \text{lower}) \, dx = \int_0^2 (2x - x^2) \, dx.$$

4. Find an antiderivative.

By the power rule, $\int 2x \, dx = x^2$ and $\int x^2 \, dx = x^3/3$.

5. Apply the bounds and simplify.

$$\begin{aligned} A &= \left[x^2 - \frac{x^3}{3} \right]_0^2 \\ &= \left(4 - \frac{8}{3} \right) - 0 = \boxed{\frac{4}{3}}. \end{aligned}$$