

Question 8

Find the average value of the function

$$f(x) = |x|$$

on the interval $[-2, 2]$.

Question 8 - Solution

The average value of a function on $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx.$$

Here $a = -2$ and $b = 2$, so

$$f_{\text{avg}} = \frac{1}{4} \int_{-2}^2 |x| \, dx.$$

Since $|x|$ is an even function,

$$\int_{-2}^2 |x| \, dx = 2 \int_0^2 x \, dx.$$

Evaluate:

$$2 \left[\frac{x^2}{2} \right]_0^2 = 2 \cdot 2 = 4.$$

Compute the average value:

$$f_{\text{avg}} = \frac{1}{4} \cdot 4 = 1.$$

1