

Question 6

Find the average value of the function

$$f(x) = \sin^2 x$$

on the interval $[0, \pi]$.

Question 6 - Solution

The average value of a function on $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx.$$

Here,

$$f_{\text{avg}} = \frac{1}{\pi} \int_0^{\pi} \sin^2 x dx.$$

Use the identity

$$\sin^2 x = \frac{1 - \cos(2x)}{2}.$$

Then

$$\int_0^{\pi} \sin^2 x dx = \frac{1}{2} \int_0^{\pi} (1 - \cos 2x) dx.$$

Integrate:

$$\frac{1}{2} \left[x - \frac{1}{2} \sin(2x) \right]_0^{\pi} = \frac{1}{2} (\pi - 0) = \frac{\pi}{2}.$$

Now divide by the interval length:

$$f_{\text{avg}} = \frac{1}{\pi} \cdot \frac{\pi}{2} = \frac{1}{2}.$$

$$\boxed{\frac{1}{2}}$$