

Question 4

Find the average value of the function

$$f(x) = \sqrt{x}(1+x)$$

on the interval $[0, 1]$.

Question 4 - Solution

The average value of a function on $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx.$$

Here,

$$f_{\text{avg}} = \int_0^1 \sqrt{x}(1+x) dx.$$

First expand the integrand:

$$\sqrt{x}(1+x) = x^{1/2} + x^{3/2}.$$

Integrate term by term:

$$\int x^{1/2} dx = \frac{2}{3}x^{3/2}, \quad \int x^{3/2} dx = \frac{2}{5}x^{5/2}.$$

Thus,

$$\int_0^1 \sqrt{x}(1+x) dx = \left[\frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} \right]_0^1.$$

Evaluate:

$$\frac{2}{3} + \frac{2}{5} = \frac{10+6}{15} = \frac{16}{15}.$$

$$\boxed{\frac{16}{15}}$$