

Question 8

Evaluate the definite integral

$$\int_0^1 \frac{2x}{(1+x^2)^2} dx.$$

Question 8 - Solution

The integrand suggests a substitution involving $1 + x^2$.

Let

$$u = 1 + x^2.$$

Then

$$du = 2x \, dx.$$

Change the limits of integration. When $x = 0$, $u = 1$. When $x = 1$, $u = 2$.

Substitute into the integral:

$$\int_0^1 \frac{2x}{(1+x^2)^2} \, dx = \int_1^2 u^{-2} \, du.$$

Integrate:

$$\int u^{-2} \, du = -u^{-1}.$$

Apply the limits:

$$-u^{-1} \Big|_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}.$$

$$\boxed{\frac{1}{2}}$$