

Question 7

Evaluate the definite integral

$$\int_1^e \frac{1}{x} \sqrt{1 + \ln x} \, dx.$$

Question 7 - Solution

The integrand suggests a substitution involving $\ln x$.

Let

$$u = 1 + \ln x.$$

Then

$$du = \frac{1}{x} dx.$$

Change the limits of integration. When $x = 1$, $u = 1$. When $x = e$, $u = 2$.

Substitute into the integral:

$$\int_1^e \frac{1}{x} \sqrt{1 + \ln x} dx = \int_1^2 \sqrt{u} du.$$

Integrate:

$$\int \sqrt{u} du = \frac{2}{3} u^{3/2}.$$

Apply the limits:

$$\left. \frac{2}{3} u^{3/2} \right|_1^2 = \frac{2}{3} (2^{3/2} - 1).$$

$$\boxed{\frac{2}{3} (2^{3/2} - 1)}$$