

Question 6

Evaluate the definite integral

$$\int_0^{\sqrt{3}} \frac{x}{1+x^2} dx.$$

Question 6 - Solution

The integrand suggests a substitution involving $1 + x^2$.

Let

$$u = 1 + x^2.$$

Then

$$du = 2x \, dx \quad \Rightarrow \quad x \, dx = \frac{1}{2} \, du.$$

Change the limits of integration. When $x = 0$, $u = 1$. When $x = \sqrt{3}$, $u = 4$.

Substitute into the integral:

$$\int_0^{\sqrt{3}} \frac{x}{1+x^2} \, dx = \frac{1}{2} \int_1^4 \frac{1}{u} \, du.$$

Integrate:

$$\frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln u.$$

Apply the limits:

$$\frac{1}{2} \ln u \Big|_1^4 = \frac{1}{2} \ln 4 = \ln 2.$$

$$\boxed{\ln 2}$$