

Question 5

Evaluate the definite integral

$$\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx.$$

Question 5 - Solution

This integral is well suited for substitution because the derivative of $\sin x$ appears in the numerator.

Let

$$u = 1 + \sin x.$$

Then

$$du = \cos x \, dx.$$

Change the limits of integration. When $x = 0$, $\sin 0 = 0$, so $u = 1$. When $x = \frac{\pi}{2}$, $\sin(\frac{\pi}{2}) = 1$, so $u = 2$.

Substitute into the integral:

$$\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} \, dx = \int_1^2 \frac{1}{u} \, du.$$

Integrate:

$$\int \frac{1}{u} \, du = \ln u.$$

Apply the limits:

$$\ln u \Big|_1^2 = \ln 2.$$

$$\boxed{\ln 2}$$