

Question 4

Evaluate the definite integral

$$\int_0^1 \frac{x^3}{\sqrt{1+x^4}} dx.$$

Question 4 - Solution

This integral suggests a substitution involving the expression inside the square root.

Let

$$u = 1 + x^4.$$

Then

$$du = 4x^3 dx \quad \Rightarrow \quad x^3 dx = \frac{1}{4} du.$$

Change the limits of integration. When $x = 0$, $u = 1$. When $x = 1$, $u = 2$.

Substitute into the integral:

$$\int_0^1 \frac{x^3}{\sqrt{1+x^4}} dx = \frac{1}{4} \int_1^2 u^{-1/2} du.$$

Integrate:

$$\frac{1}{4} \int u^{-1/2} du = \frac{1}{4} \cdot 2u^{1/2} = \frac{1}{2} \sqrt{u}.$$

Apply the limits:

$$\frac{1}{2} \sqrt{u} \Big|_1^2 = \frac{1}{2} (\sqrt{2} - 1).$$

$$\boxed{\frac{\sqrt{2} - 1}{2}}$$