

Question 3

Evaluate the definite integral

$$\int_1^2 \frac{\ln x}{x} dx.$$

Question 3 - Solution

This integral is well suited for substitution because the derivative of $\ln x$ appears in the denominator.

Let

$$u = \ln x.$$

Then

$$du = \frac{1}{x} dx.$$

Change the limits of integration. When $x = 1$, $u = \ln 1 = 0$. When $x = 2$, $u = \ln 2$.

Substitute into the integral:

$$\int_1^2 \frac{\ln x}{x} dx = \int_0^{\ln 2} u du.$$

Integrate:

$$\int u du = \frac{u^2}{2}.$$

Apply the limits:

$$\frac{u^2}{2} \Big|_0^{\ln 2} = \frac{(\ln 2)^2}{2}.$$

$$\boxed{\frac{(\ln 2)^2}{2}}$$