

Question 2

Evaluate the definite integral

$$\int_0^{\pi/2} \sin x \sqrt{1 + \cos x} \, dx.$$

Question 2 - Solution

This integral suggests a substitution involving $\cos x$ because its derivative appears in the integrand.

Let

$$u = 1 + \cos x.$$

Then

$$du = -\sin x \, dx \quad \Rightarrow \quad -du = \sin x \, dx.$$

Change the limits of integration. When $x = 0$, $\cos 0 = 1$, so $u = 2$. When $x = \frac{\pi}{2}$, $\cos(\frac{\pi}{2}) = 0$, so $u = 1$.

Substitute into the integral:

$$\int_0^{\pi/2} \sin x \sqrt{1 + \cos x} \, dx = - \int_2^1 \sqrt{u} \, du = \int_1^2 \sqrt{u} \, du.$$

Integrate:

$$\int \sqrt{u} \, du = \frac{2}{3} u^{3/2}.$$

Apply the limits:

$$\left. \frac{2}{3} u^{3/2} \right|_1^2 = \frac{2}{3} (2^{3/2} - 1).$$

$$\boxed{\frac{2}{3} (2^{3/2} - 1)}$$