

Question 10

Evaluate the definite integral

$$\int_0^1 \frac{2x}{1+x^2} dx.$$

Question 10 - Solution

This integral suggests a substitution because the derivative of the denominator appears in the numerator.

Let

$$u = 1 + x^2.$$

Then

$$du = 2x \, dx.$$

Change the limits of integration. When $x = 0$, $u = 1$. When $x = 1$, $u = 2$.

Substitute into the integral:

$$\int_0^1 \frac{2x}{1+x^2} \, dx = \int_1^2 \frac{1}{u} \, du.$$

Integrate:

$$\int \frac{1}{u} \, du = \ln u.$$

Apply the limits:

$$\ln u \Big|_1^2 = \ln 2.$$

$$\boxed{\ln 2}$$