

Question 9

Evaluate the definite integral

$$\int_1^4 \left(\frac{2}{\sqrt{x}} - \frac{1}{x} \right) dx.$$

Question 9 - Solution

Rewrite the integrand using exponents:

$$\frac{2}{\sqrt{x}} - \frac{1}{x} = 2x^{-1/2} - x^{-1}.$$

Integrate term by term:

$$\int 2x^{-1/2} dx = 2 \cdot \frac{x^{1/2}}{1/2} = 4\sqrt{x},$$

$$\int x^{-1} dx = \ln x.$$

Thus,

$$\int \left(\frac{2}{\sqrt{x}} - \frac{1}{x} \right) dx = 4\sqrt{x} - \ln x.$$

Evaluate from 1 to 4:

$$\left[4\sqrt{x} - \ln x \right]_1^4.$$

Compute the upper bound:

$$4\sqrt{4} - \ln 4 = 8 - \ln 4.$$

Compute the lower bound:

$$4\sqrt{1} - \ln 1 = 4.$$

Subtract:

$$(8 - \ln 4) - 4 = 4 - \ln 4.$$

$$\boxed{4 - \ln 4}$$