

Question 8

Evaluate the definite integral

$$\int_0^{\ln 2} e^x (1 + e^x) dx.$$

Question 8 - Solution

Begin by expanding the integrand:

$$e^x(1 + e^x) = e^x + e^{2x}.$$

Now integrate term by term:

$$\int e^x dx = e^x, \quad \int e^{2x} dx = \frac{1}{2}e^{2x}.$$

Thus,

$$\int e^x(1 + e^x) dx = e^x + \frac{1}{2}e^{2x}.$$

Evaluate from 0 to $\ln 2$:

$$\left[e^x + \frac{1}{2}e^{2x} \right]_0^{\ln 2}.$$

Compute the upper bound:

$$e^{\ln 2} + \frac{1}{2}e^{2 \ln 2} = 2 + \frac{1}{2} \cdot 4 = 4.$$

Compute the lower bound:

$$e^0 + \frac{1}{2}e^0 = 1 + \frac{1}{2} = \frac{3}{2}.$$

Subtract:

$$4 - \frac{3}{2} = \frac{5}{2}.$$

$$\boxed{\frac{5}{2}}$$