

Question 7

Evaluate the definite integral

$$\int_0^1 (3x^2 + 2)\sqrt{x} \, dx.$$

Question 7 - Solution

Begin by rewriting the integrand using powers of x :

$$(3x^2 + 2)\sqrt{x} = 3x^{5/2} + 2x^{1/2}.$$

Now integrate term by term:

$$\int 3x^{5/2} dx = 3 \cdot \frac{x^{7/2}}{7/2} = \frac{6}{7}x^{7/2},$$

$$\int 2x^{1/2} dx = 2 \cdot \frac{x^{3/2}}{3/2} = \frac{4}{3}x^{3/2}.$$

Combine the results:

$$\int (3x^2 + 2)\sqrt{x} dx = \frac{6}{7}x^{7/2} + \frac{4}{3}x^{3/2}.$$

Evaluate from 0 to 1:

$$\left[\frac{6}{7}x^{7/2} + \frac{4}{3}x^{3/2} \right]_0^1 = \frac{6}{7} + \frac{4}{3}.$$

Find a common denominator:

$$\frac{6}{7} + \frac{4}{3} = \frac{18 + 28}{21} = \frac{46}{21}.$$

$\frac{46}{21}$
