

Question 6

Evaluate the definite integral

$$\int_{-2}^2 \frac{1}{1+x^2} dx.$$

Question 6 - Solution

First observe that the function

$$\frac{1}{1+x^2}$$

is an even function, since

$$\frac{1}{1+(-x)^2} = \frac{1}{1+x^2}.$$

Therefore, the integral over the symmetric interval $[-2, 2]$ can be written as

$$\int_{-2}^2 \frac{1}{1+x^2} dx = 2 \int_0^2 \frac{1}{1+x^2} dx.$$

Now evaluate the integral. Recall that

$$\int \frac{1}{1+x^2} dx = \arctan x.$$

Thus,

$$2 \int_0^2 \frac{1}{1+x^2} dx = 2 [\arctan x]_0^2.$$

Evaluate the bounds:

$$\arctan(2) - \arctan(0) = \arctan(2).$$

Therefore,

$$\int_{-2}^2 \frac{1}{1+x^2} dx = \boxed{2 \arctan(2)}.$$