

Question 5

Evaluate the definite integral

$$\int_0^{\pi} x \sin x \, dx.$$

Question 5 - Solution

This integral suggests integration by parts because it is a product of x and $\sin x$.

Let

$$u = x, \quad dv = \sin x \, dx.$$

Then

$$du = dx, \quad v = -\cos x.$$

Apply integration by parts:

$$\int x \sin x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x.$$

Now evaluate from 0 to π :

$$[-x \cos x + \sin x]_0^\pi.$$

Compute each endpoint:

$$-\pi \cos \pi + \sin \pi = \pi,$$

$$-0 \cdot \cos 0 + \sin 0 = 0.$$

Subtract:

$$\pi - 0 = \pi.$$

$$\boxed{\pi}$$