

Question 4

Evaluate the definite integral

$$\int_0^1 \frac{x}{\sqrt{1+x^2}} dx.$$

Question 4 - Solution

This integral is well-suited for a direct substitution because the derivative of $1+x^2$ appears in the numerator.

Let

$$u = 1 + x^2.$$

Then

$$du = 2x \, dx \quad \Rightarrow \quad x \, dx = \frac{1}{2} \, du.$$

Change the limits of integration. When $x = 0$, $u = 1$. When $x = 1$, $u = 2$.

Substitute into the integral:

$$\int_0^1 \frac{x}{\sqrt{1+x^2}} \, dx = \frac{1}{2} \int_1^2 u^{-1/2} \, du.$$

Integrate:

$$\frac{1}{2} \int u^{-1/2} \, du = \frac{1}{2} \cdot 2u^{1/2} = \sqrt{u}.$$

Apply the limits:

$$\left. \sqrt{u} \right|_1^2 = \sqrt{2} - 1.$$

$$\boxed{\sqrt{2} - 1}$$