

Question 2

Evaluate the definite integral

$$\int_0^{\pi/2} (\sin x + \sin^2 x) \, dx.$$

Question 2 - Solution

Split the integral:

$$\int_0^{\pi/2} \sin x \, dx + \int_0^{\pi/2} \sin^2 x \, dx.$$

Evaluate the first integral:

$$\int_0^{\pi/2} \sin x \, dx = [-\cos x]_0^{\pi/2} = 1.$$

For the second integral, use the identity

$$\sin^2 x = \frac{1 - \cos(2x)}{2}.$$

Then

$$\int_0^{\pi/2} \sin^2 x \, dx = \frac{1}{2} \int_0^{\pi/2} (1 - \cos 2x) \, dx.$$

Integrate:

$$\frac{1}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\pi/2} = \frac{1}{2} \left(\frac{\pi}{2} - 0 \right) = \frac{\pi}{4}.$$

Combine results:

$$1 + \frac{\pi}{4}.$$

$$\boxed{1 + \frac{\pi}{4}}$$