

Question 1

Evaluate the definite integral

$$\int_0^1 (x^2 e^x - 2x e^x) \, dx.$$

Question 1 - Solution

Combine the terms:

$$\int_0^1 e^x(x^2 - 2x) dx.$$

Use integration by parts. Let

$$u = x^2 - 2x, \quad dv = e^x dx.$$

Then

$$du = (2x - 2) dx, \quad v = e^x.$$

Apply integration by parts:

$$\int e^x(x^2 - 2x) dx = (x^2 - 2x)e^x - \int e^x(2x - 2) dx.$$

Apply integration by parts again to the remaining integral. Let

$$u = 2x - 2, \quad dv = e^x dx.$$

Then

$$du = 2 dx, \quad v = e^x.$$

Thus,

$$\int e^x(2x - 2) dx = (2x - 2)e^x - \int 2e^x dx = (2x - 2)e^x - 2e^x.$$

Substitute back:

$$\int e^x(x^2 - 2x) dx = (x^2 - 2x)e^x - [(2x - 2)e^x - 2e^x].$$

Simplify:

$$(x^2 - 4x + 4)e^x.$$

Now evaluate from 0 to 1:

$$\int_0^1 e^x(x^2 - 2x) dx = [(x^2 - 4x + 4)e^x]_0^1.$$

Compute:

$$(1 - 4 + 4)e - (0 - 0 + 4) = e - 4.$$

$$\boxed{e - 4}$$