

Question 10

Let

$$f(x) = \sin x$$

on the interval $[0, \pi]$.

Using the definition of the definite integral, express

$$\int_0^{\pi} \sin x \, dx$$

as the limit of a Riemann sum using midpoints, and evaluate the limit.

Question 10 - Solution

Partition the interval $[0, \pi]$ into n equal subintervals. Each subinterval has width

$$\Delta x = \frac{\pi}{n}.$$

The midpoint of the i th subinterval is

$$x_i^* = \left(i - \frac{1}{2}\right) \Delta x = \left(i - \frac{1}{2}\right) \frac{\pi}{n}.$$

Evaluate the function at each midpoint:

$$f(x_i^*) = \sin\left(\left(i - \frac{1}{2}\right) \frac{\pi}{n}\right).$$

The Riemann sum is

$$\sum_{i=1}^n f(x_i^*) \Delta x = \frac{\pi}{n} \sum_{i=1}^n \sin\left(\left(i - \frac{1}{2}\right) \frac{\pi}{n}\right).$$

Thus,

$$\int_0^{\pi} \sin x \, dx = \lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{i=1}^n \sin\left(\left(i - \frac{1}{2}\right) \frac{\pi}{n}\right).$$

Evaluating the definite integral directly,

$$\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = (-\cos \pi) - (-\cos 0) = 2.$$

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