

Question 8

Let

$$f(x) = \frac{1}{x}$$

on the interval $[1, e]$.

Using the definition of the definite integral, express

$$\int_1^e \frac{1}{x} dx$$

as the limit of a Riemann sum using right endpoints, and evaluate the limit.

Question 8 - Solution

Partition the interval $[1, e]$ into n equal subintervals. The width of each subinterval is

$$\Delta x = \frac{e-1}{n}.$$

Using right endpoints, the i th sample point is

$$x_i = 1 + i\Delta x = 1 + i\frac{e-1}{n}.$$

Evaluate the function at each sample point:

$$f(x_i) = \frac{1}{1 + i\frac{e-1}{n}}.$$

The Riemann sum is

$$\sum_{i=1}^n f(x_i) \Delta x = \frac{e-1}{n} \sum_{i=1}^n \frac{1}{1 + i\frac{e-1}{n}}.$$

Thus,

$$\int_1^e \frac{1}{x} dx = \lim_{n \rightarrow \infty} \frac{e-1}{n} \sum_{i=1}^n \frac{1}{1 + i\frac{e-1}{n}}.$$

Evaluating the definite integral,

$$\int_1^e \frac{1}{x} dx = [\ln x]_1^e = 1.$$

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