

Question 7

Let

$$f(x) = 3 - 2x$$

on the interval $[0, 1]$.

Using the definition of the definite integral, express

$$\int_0^1 (3 - 2x) \, dx$$

as the limit of a Riemann sum using left endpoints, and evaluate the limit.

Question 7 - Solution

Partition the interval $[0, 1]$ into n equal subintervals. Each subinterval has width

$$\Delta x = \frac{1}{n}.$$

Using left endpoints, the i th sample point is

$$x_{i-1} = (i-1)\Delta x = \frac{i-1}{n}.$$

Evaluate the function at each sample point:

$$f(x_{i-1}) = 3 - 2\left(\frac{i-1}{n}\right).$$

The Riemann sum is

$$\sum_{i=1}^n f(x_{i-1}) \Delta x = \frac{1}{n} \sum_{i=1}^n \left(3 - \frac{2(i-1)}{n}\right).$$

Separate the sum:

$$\frac{1}{n} \left(\sum_{i=1}^n 3 - \frac{2}{n} \sum_{i=1}^n (i-1) \right).$$

Using

$$\sum_{i=1}^n 1 = n, \quad \sum_{i=1}^n (i-1) = \frac{n(n-1)}{2},$$

we obtain

$$\frac{1}{n} \left(3n - \frac{2}{n} \cdot \frac{n(n-1)}{2} \right) = 3 - (n-1)\frac{1}{n}.$$

Thus,

$$\int_0^1 (3-2x) dx = \lim_{n \rightarrow \infty} \left(3 - \frac{n-1}{n} \right) = 2.$$

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