

Question 6

Let

$$f(x) = |x|$$

on the interval $[-2, 2]$.

Using the definition of the definite integral, express

$$\int_{-2}^2 |x| dx$$

as the limit of a Riemann sum using equal subintervals, and evaluate the limit.

Question 6 - Solution

Divide the interval $[-2, 2]$ into n equal subintervals. Each subinterval has width

$$\Delta x = \frac{2 - (-2)}{n} = \frac{4}{n}.$$

Using right endpoints, the i th sample point is

$$x_i = -2 + i\Delta x = -2 + \frac{4i}{n}.$$

Evaluate the function:

$$f(x_i) = \left| -2 + \frac{4i}{n} \right|.$$

The Riemann sum is

$$\sum_{i=1}^n f(x_i) \Delta x = \frac{4}{n} \sum_{i=1}^n \left| -2 + \frac{4i}{n} \right|.$$

Thus,

$$\int_{-2}^2 |x| dx = \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left| -2 + \frac{4i}{n} \right|.$$

To evaluate the integral, use symmetry:

$$\int_{-2}^2 |x| dx = 2 \int_0^2 x dx.$$

Compute:

$$2 \left[\frac{x^2}{2} \right]_0^2 = 4.$$

$$\boxed{4}$$

