

Question 5

Let

$$f(x) = e^x$$

on the interval $[0, 1]$.

Using the definition of the definite integral, express

$$\int_0^1 e^x dx$$

as the limit of a Riemann sum using right endpoints, and evaluate the limit.

Question 5- Solution

Partition the interval $[0, 1]$ into n equal subintervals. Each subinterval has width

$$\Delta x = \frac{1}{n}.$$

Using right endpoints, the i th sample point is

$$x_i = i\Delta x = \frac{i}{n}.$$

Evaluate the function at each sample point:

$$f(x_i) = e^{i/n}.$$

The Riemann sum is

$$\sum_{i=1}^n f(x_i) \Delta x = \frac{1}{n} \sum_{i=1}^n e^{i/n}.$$

Thus,

$$\int_0^1 e^x dx = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n e^{i/n}.$$

Evaluating the definite integral,

$$\int_0^1 e^x dx = [e^x]_0^1 = e - 1.$$

$$\boxed{e - 1}$$

