

Question 4

Let

$$f(x) = \frac{1}{1+x^2}$$

on the interval $[-1, 1]$.

Using the definition of the definite integral, express

$$\int_{-1}^1 \frac{1}{1+x^2} dx$$

as the limit of a Riemann sum using right endpoints, and evaluate the limit.

Question 4 - Solution

Divide the interval $[-1, 1]$ into n equal subintervals. The width of each subinterval is

$$\Delta x = \frac{1 - (-1)}{n} = \frac{2}{n}.$$

Using right endpoints, the i th sample point is

$$x_i = -1 + i\Delta x = -1 + \frac{2i}{n}.$$

Evaluate the function:

$$f(x_i) = \frac{1}{1 + \left(-1 + \frac{2i}{n}\right)^2}.$$

The Riemann sum is

$$\sum_{i=1}^n f(x_i) \Delta x = \frac{2}{n} \sum_{i=1}^n \frac{1}{1 + \left(-1 + \frac{2i}{n}\right)^2}.$$

Thus,

$$\int_{-1}^1 \frac{1}{1+x^2} dx = \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \frac{1}{1 + \left(-1 + \frac{2i}{n}\right)^2}.$$

Evaluating the definite integral,

$$\int_{-1}^1 \frac{1}{1+x^2} dx = [\arctan x]_{-1}^1 = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}.$$

$$\boxed{\frac{\pi}{2}}$$

