

Question 3

Let

$$f(x) = \sqrt{x}$$

on the interval $[0, 4]$.

Using the definition of the definite integral, write

$$\int_0^4 \sqrt{x} \, dx$$

as the limit of a Riemann sum using left endpoints, and evaluate the limit.

Question 3 - Solution

Divide the interval $[0, 4]$ into n equal subintervals. Each subinterval has width

$$\Delta x = \frac{4}{n}.$$

Using left endpoints, the i th sample point is

$$x_{i-1} = (i-1)\Delta x = \frac{4(i-1)}{n}.$$

Evaluate the function:

$$f(x_{i-1}) = \sqrt{\frac{4(i-1)}{n}} = 2\sqrt{\frac{i-1}{n}}.$$

The Riemann sum is

$$\sum_{i=1}^n f(x_{i-1}) \Delta x = \sum_{i=1}^n 2\sqrt{\frac{i-1}{n}} \cdot \frac{4}{n} = \frac{8}{n^{3/2}} \sum_{i=1}^n \sqrt{i-1}.$$

Thus,

$$\int_0^4 \sqrt{x} \, dx = \lim_{n \rightarrow \infty} \frac{8}{n^{3/2}} \sum_{i=1}^n \sqrt{i-1}.$$

As $n \rightarrow \infty$, this Riemann sum converges to the definite integral:

$$\int_0^4 \sqrt{x} \, dx = \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{16}{3}.$$

$$\boxed{\frac{16}{3}}$$

