

Question 2

Let

$$f(x) = x^2$$

on the interval $[1, 3]$.

Using the definition of the definite integral, write

$$\int_1^3 x^2 dx$$

as the limit of a Riemann sum using midpoints.

Question 2 - Solution

Divide $[1, 3]$ into n equal subintervals of width $\Delta x = 2/n$.

The midpoints are $x_i^* = 1 + (2i - 1)/n$.

$$M_n = \frac{2}{n} \sum_{i=1}^n \left(1 + \frac{2i-1}{n} \right)^2.$$

Use $\sum (2i - 1) = n^2$ and $\sum (2i - 1)^2 = n(4n^2 - 1)/3$:

$$M_n = 2 + 4 + \frac{2(4n^2 - 1)}{3n^2} = \frac{26}{3} - \frac{2}{3n^2}.$$

$$\boxed{\int_1^3 x^2 dx = \lim_{n \rightarrow \infty} M_n = \frac{26}{3}.$$