

Question 1

Consider the function

$$f(x) = 2x + 1$$

on the interval $[0, 2]$.

Using the definition of the definite integral, express

$$\int_0^2 (2x + 1) \, dx$$

as the limit of a Riemann sum with n equal subintervals.

Question 1 - Solution

Divide the interval $[0, 2]$ into n equal subintervals. Each subinterval has width

$$\Delta x = \frac{2}{n}.$$

Using right endpoints, the i th sample point is

$$x_i = 0 + i\Delta x = \frac{2i}{n}.$$

Evaluate the function at x_i :

$$f(x_i) = 2\left(\frac{2i}{n}\right) + 1 = \frac{4i}{n} + 1.$$

The Riemann sum is

$$\sum_{i=1}^n f(x_i) \Delta x = \sum_{i=1}^n \left(\frac{4i}{n} + 1\right) \frac{2}{n}.$$

Simplify:

$$\frac{2}{n} \sum_{i=1}^n \left(\frac{4i}{n} + 1\right) = \frac{8}{n^2} \sum_{i=1}^n i + \frac{2}{n} \sum_{i=1}^n 1.$$

Using

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n 1 = n,$$

we obtain

$$\frac{8}{n^2} \cdot \frac{n(n+1)}{2} + \frac{2}{n} \cdot n = 4\left(1 + \frac{1}{n}\right) + 2.$$

Thus,

$$\int_0^2 (2x + 1) dx = \lim_{n \rightarrow \infty} \left(6 + \frac{4}{n}\right) = 6.$$

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