

Question 10

Find the exact area of the region bounded by the curves

$$y = \sqrt{x} \quad \text{and} \quad y = \ln x,$$

between $x = 1$ and $x = e^2$.

Question 10 - Solution

On $1 \leq x \leq e^2$, we have $\sqrt{x} \geq \ln x$, so

$$A = \int_1^{e^2} (\sqrt{x} - \ln x) dx.$$

Compute:

$$\int_1^{e^2} \sqrt{x} dx = \left[\frac{2}{3} x^{3/2} \right]_1^{e^2} = \frac{2}{3} (e^3 - 1),$$

$$\int_1^{e^2} \ln x dx = [x \ln x - x]_1^{e^2} = e^2 + 1.$$

Thus

$$A = \frac{2}{3} (e^3 - 1) - (e^2 + 1) = \frac{2}{3} e^3 - e^2 - \frac{5}{3}.$$

$$\boxed{\frac{2}{3} e^3 - e^2 - \frac{5}{3}}$$

