

Question 7

Find the exact area of the region enclosed by the curves

$$y = \sin x \quad \text{and} \quad y = \cos x$$

on the interval

$$\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}.$$

Question 7 - Solution

First find the points of intersection by solving

$$\sin x = \cos x.$$

This gives

$$\tan x = 1 \quad \Rightarrow \quad x = \frac{\pi}{4} + k\pi.$$

The smallest two consecutive intersection points occur at

$$x = \frac{\pi}{4} \quad \text{and} \quad x = \frac{5\pi}{4}.$$

On the interval between these points, we have

$$\sin x \geq \cos x.$$

Thus the enclosed area is

$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx.$$

Compute:

$$\int \sin x \, dx = -\cos x, \quad \int \cos x \, dx = \sin x.$$

So

$$A = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}.$$

Evaluate:

$$\begin{aligned} -\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right) &= \sqrt{2}, \\ -\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) &= -\sqrt{2}. \end{aligned}$$

Thus,

$$A = \sqrt{2} - (-\sqrt{2}) = 2\sqrt{2}.$$

$$\boxed{2\sqrt{2}}$$

