

Question 6

Find the exact area of the region enclosed by the curves

$$y = x \quad \text{and} \quad y = x^3.$$

Question 6 - Solution

First find the points of intersection:

$$x = x^3 \Rightarrow x(x^2 - 1) = 0.$$

Thus,

$$x = -1, 0, 1.$$

On the interval $(0, 1)$, we have

$$x \geq x^3,$$

and by symmetry, the same configuration occurs on $(-1, 0)$.

Therefore, the total enclosed area is twice the area on $[0, 1]$:

$$A = 2 \int_0^1 (x - x^3) dx.$$

Evaluate the integral:

$$\int_0^1 (x - x^3) dx = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Thus,

$$A = 2 \cdot \frac{1}{4} = \frac{1}{2}.$$

$$\boxed{\frac{1}{2}}$$

