

Question 5

Find the exact area of the region enclosed by the curves

$$y = |x| \quad \text{and} \quad y = x^2.$$

Question 5 - Solution

First find the points of intersection by solving

$$|x| = x^2.$$

For $x \geq 0$, this becomes

$$x = x^2 \Rightarrow x = 0 \text{ or } x = 1.$$

For $x < 0$, we have

$$-x = x^2 \Rightarrow x = 0 \text{ or } x = -1.$$

Thus, the curves intersect at

$$x = -1, 0, 1.$$

On $[0, 1]$, we have $|x| = x \geq x^2$, and by symmetry the same holds on $[-1, 0]$. Therefore, the total area is twice the area on $[0, 1]$:

$$A = 2 \int_0^1 (x - x^2) dx.$$

Compute the integral:

$$\int_0^1 (x - x^2) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

Thus,

$$A = 2 \cdot \frac{1}{6} = \frac{1}{3}.$$

$$\boxed{\frac{1}{3}}$$

