

Question 4

Find the exact area of the region enclosed by the curves

$$y = x\sqrt{2-x} \quad \text{and} \quad y = x.$$

Question 4 - Solution

The intersections satisfy $x\sqrt{2-x} = x$, giving $x = 0, 1$.

For $0 < x < 1$, the square-root curve is above the line, hence

$$A = \int_0^1 (x\sqrt{2-x} - x) dx.$$

With $u = 2 - x$,

$$\int_0^1 x\sqrt{2-x} dx = \int_1^2 (2-u)\sqrt{u} du = \left[\frac{4}{3}u^{3/2} - \frac{2}{5}u^{5/2} \right]_1^2 = \frac{16\sqrt{2} - 14}{15}.$$

$$A = \frac{32\sqrt{2} - 43}{30} > 0.$$