

Question 3

Find the exact area of the region enclosed by the curves

$$y = \sqrt{x} \quad \text{and} \quad y = x^2,$$

restricted to the interval $0 \leq x \leq 1$.

Question 3 - Solution

First determine which curve lies above the other on $[0, 1]$. For $0 < x < 1$,

$$\sqrt{x} \geq x^2.$$

Thus, the enclosed area is

$$A = \int_0^1 (\sqrt{x} - x^2) dx.$$

Evaluate each term:

$$\int_0^1 \sqrt{x} dx = \left[\frac{2}{3} x^{3/2} \right]_0^1 = \frac{2}{3},$$

$$\int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3}.$$

Subtract:

$$A = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

$$\boxed{\frac{1}{3}}$$

