

Question 2

Find the exact area of the region enclosed by the curves

$$y = \ln x \quad \text{and} \quad y = x - 1,$$

restricted to the interval $0 < x \leq 1$.

Question 2 - Solution

On the interval $0 < x \leq 1$, the curves intersect where

$$\ln x = x - 1,$$

which occurs at

$$x = 1.$$

For $0 < x < 1$, we have

$$x - 1 \geq \ln x,$$

so the area between the curves is

$$A = \int_0^1 ((x - 1) - \ln x) dx.$$

Compute each term:

$$\int_0^1 (x - 1) dx = \left[\frac{x^2}{2} - x \right]_0^1 = -\frac{1}{2},$$

$$\int_0^1 \ln x dx = [x \ln x - x]_0^1 = -1.$$

Therefore,

$$A = -\frac{1}{2} - (-1) = \frac{1}{2}.$$

$$\boxed{\frac{1}{2}}$$

