

Question 9

Evaluate the integral

$$\int \frac{x+1}{(x^2+2x+3)^{3/2}} dx.$$

Question 9 - Solution

Begin by completing the square in the denominator:

$$x^2 + 2x + 3 = (x + 1)^2 + 2.$$

Let

$$u = x + 1.$$

Then

$$du = dx, \quad (x^2 + 2x + 3)^{3/2} = (u^2 + 2)^{3/2}.$$

Substitute into the integral:

$$\int \frac{u}{(u^2 + 2)^{3/2}} du.$$

Now use a direct substitution. Let

$$w = u^2 + 2, \quad dw = 2u du.$$

Then

$$\int \frac{u}{(u^2 + 2)^{3/2}} du = \frac{1}{2} \int w^{-3/2} dw.$$

Integrate:

$$\frac{1}{2} \int w^{-3/2} dw = \frac{1}{2} \left(-2w^{-1/2} \right) = -\frac{1}{\sqrt{w}}.$$

Substitute back:

$$\boxed{-\frac{1}{\sqrt{x^2 + 2x + 3}} + C}$$