

Question 8

Evaluate the integral

$$\int \frac{x+2}{\sqrt{x^2+4x+8}} dx.$$

Question 8 - Solution

Complete the square in the denominator:

$$x^2 + 4x + 8 = (x + 2)^2 + 4.$$

Let

$$u = x + 2.$$

Then

$$du = dx, \quad x + 2 = u.$$

Substitute into the integral:

$$\int \frac{x + 2}{\sqrt{x^2 + 4x + 8}} dx = \int \frac{u}{\sqrt{u^2 + 4}} du.$$

Now use a direct substitution. Let

$$w = u^2 + 4, \quad dw = 2u du.$$

Then

$$\int \frac{u}{\sqrt{u^2 + 4}} du = \frac{1}{2} \int w^{-1/2} dw = \sqrt{w}.$$

Substitute back:

$$\boxed{\sqrt{x^2 + 4x + 8} + C}$$