

Question 10

Evaluate the integral

$$\int x \ln(1 - x^2) dx.$$

Question 10 - Solution

Let

$$u = 1 - x^2.$$

Then

$$du = -2x \, dx.$$

Rewrite the integral:

$$\int x \ln(1 - x^2) \, dx = -\frac{1}{2} \int \ln u \, du.$$

Now integrate:

$$\int \ln u \, du = u \ln u - u.$$

Therefore,

$$-\frac{1}{2}(u \ln u - u) + C.$$

Substitute back:

$$-\frac{1}{2}[(1 - x^2) \ln(1 - x^2) - (1 - x^2)] + C$$