

Question 9

Evaluate the integral

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx.$$

Question 9 - Solution

Rewrite the integrand to reveal a derivative pattern:

$$2 \cos(\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x}).$$

Let

$$u = \sqrt{x}.$$

Then

$$du = \frac{1}{2\sqrt{x}} dx \quad \Rightarrow \quad dx = 2\sqrt{x} du.$$

Substitute into the integral:

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = \int \frac{\cos(u)}{\sqrt{x}} \cdot 2\sqrt{x} du = 2 \int \cos u du.$$

Integrate:

$$2 \int \cos u du = 2 \sin u.$$

Substitute back:

$$\boxed{2 \sin(\sqrt{x}) + C}$$