

Question 6

Evaluate the integral

$$\int \frac{x e^{1/(1+x^2)}}{(1+x^2)^2} dx.$$

Question 6 - Solution

Use substitution. Let

$$u = \frac{1}{1+x^2}.$$

Then

$$du = -\frac{2x}{(1+x^2)^2} dx \quad \Rightarrow \quad -\frac{1}{2} du = \frac{x}{(1+x^2)^2} dx.$$

Substitute into the integral:

$$\int \frac{x e^{1/(1+x^2)}}{(1+x^2)^2} dx = -\frac{1}{2} \int e^u du.$$

Integrate:

$$-\frac{1}{2} \int e^u du = -\frac{1}{2} e^u.$$

Substitute back:

$$\boxed{-\frac{1}{2} e^{1/(1+x^2)} + C}$$