

Question 5

Evaluate the integral

$$\int \frac{2x}{1 + \sqrt{1 + x^2}} dx.$$

Question 5 - Solution

Use substitution. Let

$$u = \sqrt{1 + x^2}.$$

Then

$$u^2 = 1 + x^2 \quad \Rightarrow \quad 2u \, du = 2x \, dx \quad \Rightarrow \quad 2x \, dx = 2u \, du.$$

Substitute into the integral:

$$\int \frac{2x}{1 + \sqrt{1 + x^2}} \, dx = \int \frac{2u}{1 + u} \, du.$$

Rewrite the integrand:

$$\frac{2u}{1 + u} = 2 \left(1 - \frac{1}{1 + u} \right).$$

Integrate:

$$\int 2 \left(1 - \frac{1}{1 + u} \right) \, du = 2u - 2 \ln(1 + u).$$

Substitute back:

$$\boxed{2\sqrt{1 + x^2} - 2 \ln(1 + \sqrt{1 + x^2}) + C}$$