

Question 4

Evaluate the integral

$$\int \frac{x \sin(x^2)}{\sqrt{1 + \cos(x^2)}} dx.$$

Question 4 - Solution

Use substitution. Let

$$u = \cos(x^2).$$

Then

$$du = -\sin(x^2) \cdot 2x \, dx \quad \Rightarrow \quad -\frac{1}{2} du = x \sin(x^2) \, dx.$$

Substitute into the integral:

$$\int \frac{x \sin(x^2)}{\sqrt{1 + \cos(x^2)}} \, dx = -\frac{1}{2} \int \frac{du}{\sqrt{1 + u}}.$$

Integrate:

$$-\frac{1}{2} \int (1 + u)^{-1/2} \, du = -\frac{1}{2} \cdot 2(1 + u)^{1/2} = -\sqrt{1 + u}.$$

Substitute back:

$$-\sqrt{1 + \cos(x^2)} + C$$