

Question 3

Evaluate the integral

$$\int \frac{x^3}{\sqrt{1+x^4}} dx.$$

Question 3 - Solution

Use substitution. Let

$$u = 1 + x^4.$$

Then

$$du = 4x^3 dx \quad \Rightarrow \quad x^3 dx = \frac{1}{4} du.$$

Substitute into the integral:

$$\int \frac{x^3}{\sqrt{1+x^4}} dx = \frac{1}{4} \int u^{-1/2} du.$$

Integrate:

$$\frac{1}{4} \int u^{-1/2} du = \frac{1}{4} \cdot 2u^{1/2} = \frac{1}{2} \sqrt{u}.$$

Substitute back:

$$\boxed{\frac{1}{2} \sqrt{1+x^4} + C}$$