

Question 1

Evaluate the integral

$$\int \frac{x e^{\sqrt{1+x^2}}}{\sqrt{1+x^2}} dx.$$

Question 1 - Solution

Use substitution. Let

$$u = \sqrt{1+x^2}.$$

Then

$$u^2 = 1 + x^2 \quad \Rightarrow \quad 2u \, du = 2x \, dx \quad \Rightarrow \quad x \, dx = u \, du.$$

Substitute into the integral:

$$\int \frac{x e^{\sqrt{1+x^2}}}{\sqrt{1+x^2}} \, dx = \int \frac{u e^u}{u} \, du = \int e^u \, du.$$

Integrate:

$$\int e^u \, du = e^u.$$

Substitute back:

$$\boxed{e^{\sqrt{1+x^2}} + C}$$