

**Question 10**

Evaluate the indefinite integral:

$$\int \frac{x^2 + 2x + 2}{\sqrt{x^2 + 2x + 5}} dx.$$

## Question 10 - Solution

We are given:

$$\int \frac{x^2 + 2x + 2}{\sqrt{x^2 + 2x + 5}} dx$$

Rewrite the numerator:

$$x^2 + 2x + 2 = (x^2 + 2x + 5) - 3.$$

Then

$$\int \frac{x^2 + 2x + 2}{\sqrt{x^2 + 2x + 5}} dx = \int \sqrt{x^2 + 2x + 5} dx - 3 \int \frac{dx}{\sqrt{x^2 + 2x + 5}}.$$

Complete the square:

$$x^2 + 2x + 5 = (x + 1)^2 + 4.$$

Let

$$t = x + 1.$$

Then

$$x^2 + 2x + 5 = t^2 + 4$$

and

$$dt = dx.$$

So the integral becomes:

$$\int \sqrt{t^2 + 4} dt - 3 \int \frac{dt}{\sqrt{t^2 + 4}}.$$

Using the formulas

$$\int \sqrt{t^2 + a^2} dt = \frac{t}{2} \sqrt{t^2 + a^2} + \frac{a^2}{2} \ln |t + \sqrt{t^2 + a^2}|$$

and

$$\int \frac{dt}{\sqrt{t^2 + a^2}} = \ln |t + \sqrt{t^2 + a^2}|,$$

with  $a = 2$ , we get:

$$\int \sqrt{t^2 + 4} dt = \frac{t}{2} \sqrt{t^2 + 4} + 2 \ln |t + \sqrt{t^2 + 4}|$$

and

$$3 \int \frac{dt}{\sqrt{t^2 + 4}} = 3 \ln |t + \sqrt{t^2 + 4}|.$$

Therefore,

$$\int \frac{x^2 + 2x + 2}{\sqrt{x^2 + 2x + 5}} dx = \frac{t}{2} \sqrt{t^2 + 4} + 2 \ln |t + \sqrt{t^2 + 4}| - 3 \ln |t + \sqrt{t^2 + 4}|.$$

Simplify:

$$= \frac{t}{2} \sqrt{t^2 + 4} - \ln |t + \sqrt{t^2 + 4}| + C.$$

Substitute back  $t = x + 1$ :

$$\boxed{\frac{x+1}{2} \sqrt{x^2 + 2x + 5} - \ln |x + 1 + \sqrt{x^2 + 2x + 5}| + C}$$