

Question 7

Evaluate the integral

$$\int \frac{\sqrt{x^2 + 1}}{x} dx.$$

Question 7 - Solution

For $x \neq 0$, write $s = \sqrt{1+x^2}$, so $ds = (x/s) dx$ and $x^2 = s^2 - 1$.

$$\int \frac{\sqrt{1+x^2}}{x} dx = \int \frac{s^2}{s^2-1} ds = s + \frac{1}{2} \ln \left| \frac{s-1}{s+1} \right| + C.$$

Since $(s-1)(s+1) = x^2$, this becomes

$$\boxed{\sqrt{1+x^2} + \ln \left| \frac{x}{1+\sqrt{1+x^2}} \right| + C.}$$

The formula is valid on either interval $(-\infty, 0)$ or $(0, \infty)$; constants may differ between intervals.